INTERPOLATION ALGORITHMS USABLE AT THE CONTOURING SIMULATION ON THE CNC MACHINE-TOOLS

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Abstract: The machining accuracy in a CNC machine tool greatly depends on the interpolation algorithm which consists of the interpolator and the acceleration/deceleration algorithm. In the conventional interpolation algorithm, there is a path error between the desired curve generated by the interpolator and the coordinated one from the accelerated/decelerated traveling distances along the X, Y, and Z axes.
In this paper, algorithms are proposed for the generation of the CNC machine-tool interpolation control kinematical parameters, usable in the simulation of the CNC machine-tool contouring motions on models.

1. INTRODUCTION

The in-time correlation of the motions on the numerically controlled machine axes in the contouring stage involves the successive control of a very great number of points that define the part profile. This big quantity of data can be introduced through program in the numerical control equipment by means of some proper program carriers, but the solution is practically a non advisable one. Another variant resides in the getting of the points, which define the path, following to some calculations made by an own unit within the numerical control equipment. This method that is generally applied today will constitute the single solution that will be further used in the model of the CNC lathe numerical control simulation.

In practice, it is mainly necessary to be made interpolation for the straight line and the circle, the most usual curves. The most parts have profiles that can be divided in these two types of curves. Some necessities for a parabola or ellipse interpolation seldom appear and in very special cases for curves that can't be defined by an explicit mathematical equation and are determined by a number of points. In this last case, the role of the interpolator is to provide the coordinates of the intermediate curve points on the basis of some formula for their approximation.

The main characteristic of such systems is the discrete running given by the digital character of the elements from their structure. The values that are used in system are estimated and adapted at equal discrete time intervals, these intervals being named sample periods.

The programmed motion is reached by the control of a certain profile for the acceleration of each numerical axis servomotor. The graph of the servomotor aceleration variation during the programmed motion execution, assumes a T_c long cycle consisted of the acceleration time, T_1, the time of null acceleration (constant velocity), T_2, and the deceleration time that results from the subtraction $[T_c - (T_1 + T_2)]$.

The main industrial movement laws are the rectangular acceleration profile (bang-bang command) and trapezoidal acceleration profile (limited jerk command), both presented in Figure 1.

The first kind of profile makes it possible to carry out a movement easily without speed discontinuity; on the other hand acceleration is piecewise continuous. This implies acceleration stresses, also called jerks, which strongly contribute to the preponderance of

the vibratory terms in the dynamical error [2]. The acceleration profile with controlled jerk offers a continuous acceleration.
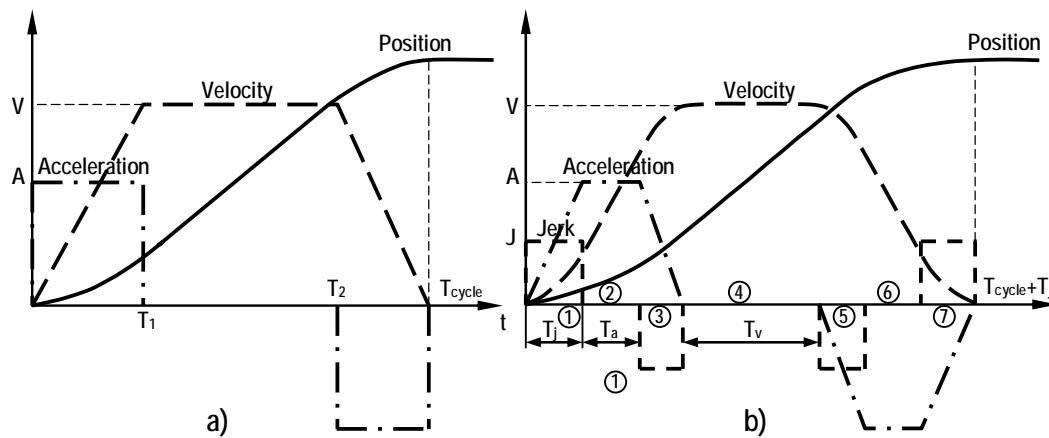

Figure 1. Classical movement laws:
(a) rectangular acceleration profile and (b) trapezoidal acceleration profile (limited jerk profile)

A jerk is defined as, [5],:

$$j = \frac{\mathrm{d}a}{\mathrm{d}t}$$ (1)

For a constant jerk A, and a final acceleration value J, we obtain for the jerk limitation time:

$$T_j = \frac{A}{J}$$ (2)

However, it lengthens the movement time by one constant jerk time (denoted T_j in figure 1) for the same value of maximum acceleration value. Intuitively, the weaker the value of jerk is, the softer the movement law, and, finally, the less significant the amplitudes of the vibrations induced by this law are, [2].

The profile generation of the kinematical parameters for the CNC machine-tool movement controls is performed by the CNC machine-tool computer, on the basis of some software algorithms.

The interpolation algorithm running has as result the generation of the reference impulses for each machine axis, at time intervals equal with the sample period. Each impulse has as result the generation of an input ramp type signal at the input of the closed loop position servo-system, the final path being resulted from the combination in plane (or in space) of the numerical axis responses at these ramp type inputs.

In this paper, algorithms are proposed for the generation of the control kinematical parameters in order that some paths typical to the numerically controlled machine-tool contouring stage to be performed (the straight line and the circle), for the both above mentioned variants of movement laws. The proposed algorithms can be used in the simulation of the contouring motions [4] on models, for the appraisal of their accuracy performances in the preliminary designing phase.

2. ALGORITHM OF THE KINEMATICAL PARAMETER GENERATION FOR THE RECTANGULAR ACCELERATION PROFILE

In the model proposed for simulation, the rectangular acceleration profile is a symmetrical one, the acceleration and deceleration phases having the same long in time and the same value of the acceleration, a, a positive value for the acceleration phase and

a negative value for the deceleration phase.

The meanings of the used values are, as follows:

v – programmed feed velocity, in [m/s];

a – programmed acceleration for the acceleration phases, in [m/s^2];

T_1 – duration of acceleration and deceleration segments, in [s];

T_2 – duration of constant velocity segment, in [s];

N_1 – number of interpolation steps (elementary increments) for the acceleration and deceleration phases;

N_2 – number of interpolation steps (elementary increments) for the constant velocity phase;

t_1 – set of the interpolation points on the acceleration segments, in [s];

t_2 – set of the interpolation points on the constant velocity segment, in [s];

T_a – dwell time for the complete performance of the controlled motions, in [s]; $T_a = 0.5$ [s];

N_a – number of interpolation steps (elementary increments) for the dwell period, T_a;

t_a – set of the interpolation points from the dwell period, T_a ;

T_s – sample period, in [s]; $T_s = 0.012$ [s];

s_1 – distance traveled on path in the acceleration and the deceleration phases, in [m];

s_2 – distance traveled on path in the constant velocity phase, in [m].

The relations for the calculus of the kinematical parameters: space, velocity and acceleration, are those known from the rectilinear motion kinematics. The number of the interpolation steps for each phase, are obtained by dividing the initial phase calculated time, T_i, at the sample period, T_s, rounded to the nearest integer towards $+\infty$:

$$N_i = \text{round} \frac{T_i}{T_s} \qquad (3)$$

The interpolator model was built using the facilities of the Matlab medium, which has typical functions for the generation of the set of points which are the limit of the interpolation intervals, using the matrix operation theory.

As the model performances to be relieved, further on, the results of the interpolation simulation in plane of a straight line segment with 100 mm length and having a 15° angle related to the one of the coordinate system axes, are presented. The initial kinematical data are: the velocity $v = 10$ [m/min] and the acceleration $a = 1$ [m/s^2].
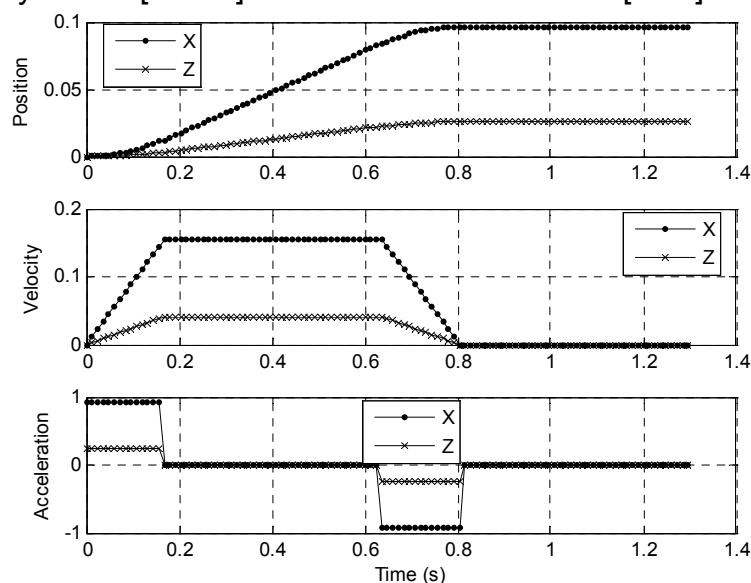
Figure 2. Generation of the control kinematical parameters for a system with two axes (typical to the CNC lathes) The controlled motion: interpolation after a straight path in plane

3. ALGORITHM OF THE KINEMATICAL PARAMETER GENERATION FOR THE TRAPEZOIDAL ACCELERATION PROFILE

Although it is easy to be practically implemented, the trapezoidal acceleration profile has the disadvantage of the sudden acceleration variation that being very important especially for the contouring manufacturing. The sudden variation of the acceleration generates the phenomenon named jerk, which has a negative influence upon the feed system behavior, especially for the contouring manufacturing. The jerk determines not only the influence upon the accuracy of the performed profile, but also the oscillation introduction in system. For this reason, the jerk limitation is an important method for the improvement of the dynamic numerically controlled machine-tool behavior, respectively the programmed profile performance accuracy, [2], [3], [6].

In this content, and the algorithm of the kinematical profile generation was developed, being briefly below shown.

According to the indications from Figure 1, the algorithm initialization assumes the definition of the initial and final values for position, feed velocity, v, maximum limit of acceleration, A, maximum limit of deceleration, D, and jerk limit, J.

The jerk limit is settled from the maximum acceleration, A, divided by the acceleration time, J, (equation (2)). The functions of the kinematical parameter variation on the intervals 1, 2, 3 and 4 (Figure 1) are:

$$s_1 = \frac{1}{6} J t_1^3 ; \quad v_1 = \frac{1}{2} J t_1^2 ; \quad a_1 = J t_1 \text{ - for the interval 1;} \tag{4}$$

$$s_2 = s_1 + v_1 t_2 + \frac{1}{2} A t_2^2 ; \quad v_2 = v_1 + A t_2 ; \quad a_2 = A \text{ - for the interval 2;} \tag{5}$$

$$s_3 = s_2 + v_2 t_3 + \frac{1}{2} A t_3^2 - \frac{1}{6} J t_3^3 ; \quad v_3 = v_2 + A t_3 - \frac{1}{2} J t_3^2 ; \quad a_3 = A - J t_3 \text{ - for the interval 3;} \tag{6}$$

$$s_4 = s_3 + v_3 t_4 ; \quad v_4 = v_3 ; \quad a_4 = 0 \text{ - for the interval 4.} \tag{7}$$

For the others 3 intervals of the cycle, the functions of the kinematical parameter variations are anti-symmetrical related to those of the intervals 1, 2 and 3, if the deceleration value is the same with the acceleration value:

$$s_5 = s_4 + v_4 t_5 - \frac{1}{6} J t_5^3 ; \quad v_5 = v_4 = v_3 ; \quad a_5 = J t_5 \text{ - for the interval 5;} \tag{8}$$

$$s_6 = s_5 + v_5 t_6 - \frac{1}{2} A t_6^2 ; \quad v_6 = v_5 - A t_6 ; \quad a_6 = -A \text{ - for the interval 6;} \tag{9}$$

$$s_7 = s_6 - v_6 t_7 - \frac{1}{2} A t_7^2 - \frac{1}{6} J t_7^3 ; \quad v_7 = v_6 + A t_7 - \frac{1}{2} J t_7^2 ; \quad a_7 = -A + J t_7 \text{ - for the interval 7;} \tag{10}$$

Before the generation of the incremental position controls in the interpolation stage, for the kinematical limited jerk profile shown in the figure 1 to be implemented, must be firstly determined the number of the interpolation steps and the time interval between them. The number of the interpolation steps for each phase is obtained by dividing the initial calculated phase time, T_i, at the sample period, T_s, rounded to the nearest integer towards $+\infty$, according to the equation (3).

Further on, the study presents the performances offered by the interpolation algorithm with trapezoidal acceleration profile at the contouring motion of a circle with a diameter of 10 [mm], typical motion of a CNC milling machine. As the circle generation to be performed with constant velocity, the acceleration and deceleration phases of the

kinematical cycle will be performed on rectilinear paths, tangents to circle (Figure 3). In the Figure 4 is restored the velocity profile generated by the above described interpolation algorithm, with the sample period adjusted at 12 [ms].
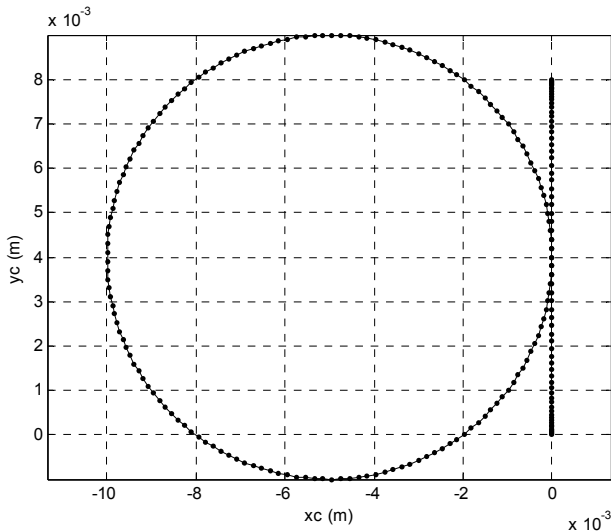

**Figure 3. Path of the interpolation profile: circle of 10 [mm] diameter**
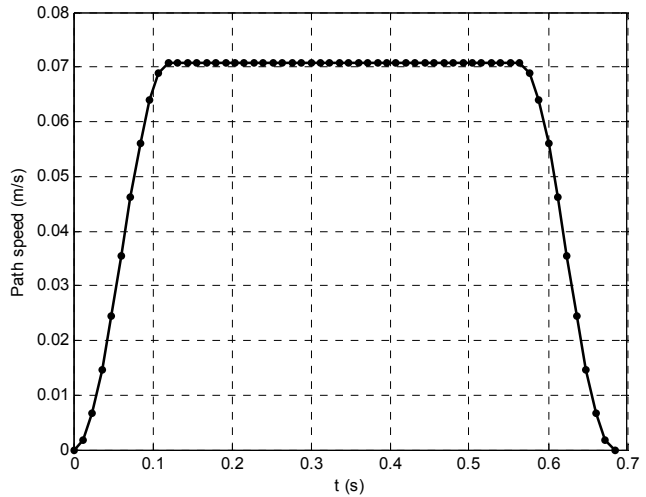
Figure 4. Velocity profile generated by the interpolator for v_{max} = 0.0707 [m/s]; $A = a_{max}$ = 1 [m/s²]; J = 59.5 [m/s³];

In the Figures 5 and 6 are shown the graphs of the kinematical parameter variation for the servomotors of those two numerical axes, their motion combination being that which generates the circular path presented in the Figure 3.
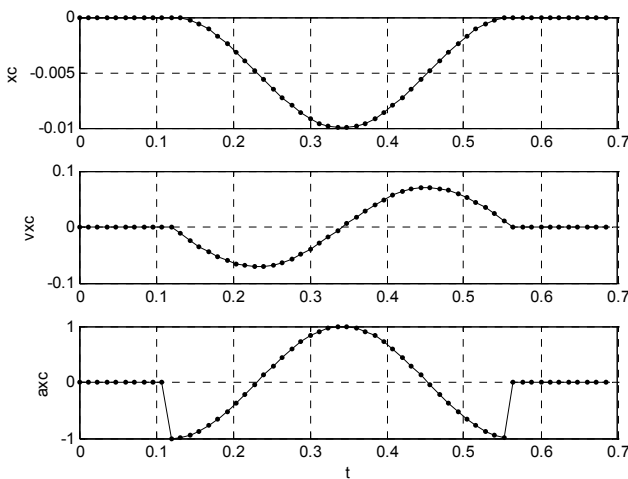
Figure 5. Variation graphs of the kinematical parameters (distance, velocity, acceleration) generated by the interpolator for the numerical X axis
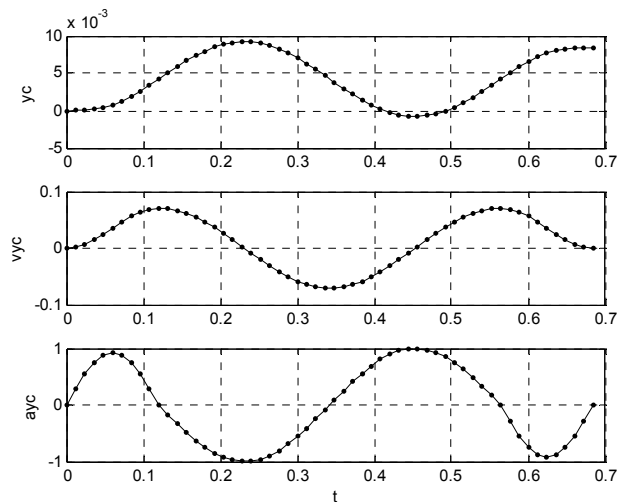
Figure 6. Variation graphs of the kinematical parameters (distance, velocity, acceleration) generated by the interpolator for the numerical Y axis

If the dimension of the interpolation circular path is kept and the initial data are changed, respectively the maximum velocity, the maximum acceleration and the jerk limit, will be also clearly changed the variation graphs of the kinematical parameters generated by the interpolator, as well as the number of the interpolation intervals (the sample period being kept constant, this being dictated by the position encoder resolution) and all these can be seen in the Figure 7.
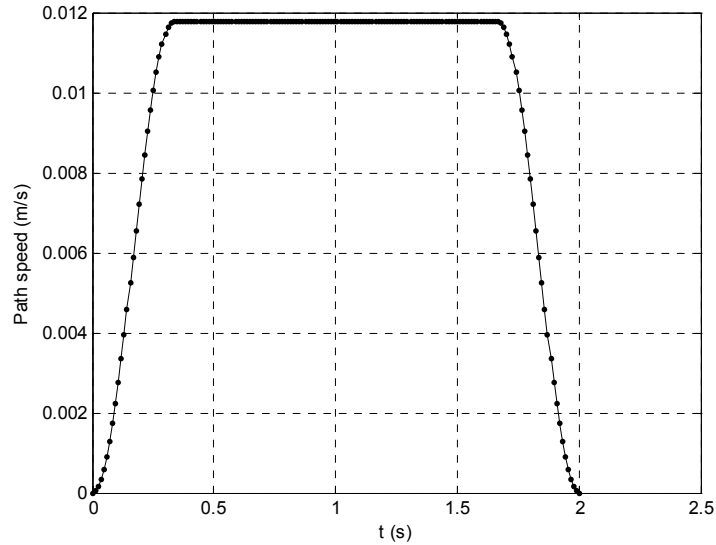
Figure 7. Velocity profile generated by the interpolator for v_{max} = 0.0118 [m/s];
A = a_{max} = 0.0556 [m/s^2]; J = 4.11 [m/s^3];

4. CONCLUSIONS

The modern tackling in the field of the CNC machine-tools designing, assumes the usage of the mechatronic syntheses of analysis and optimization, especially in the preliminary designing phase, when more possible variants must be analyzed in a short time for the optimum solution finding. For this, the models of the machine assembly behavior simulation are necessary, the machine assembly being consisted of structure and control system with an increasingly level complexity because of the progress in the field of the engineering calculus software tools.

In the present paper software interpolation algorithms are proposed for two acceleration variants of the movement classic laws: rectangular acceleration profile and trapezoidal acceleration profile (with limited jerk).

As the performances of the proposed algorithms to be illustrated, the case of the kinematical parameter generation is presented at the linear interpolation and the circular interpolation.

The algorithms are useful in the simulation of the CNC machine-tool motions on models, for the optimization of the parameters, which can improve the dynamic behavior and the contouring accuracy.

5. REFERENCES

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